Evaluation of a Capacitance Scaling System

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Abstract—**An improved error analysis of an existing capacitance scaling system for supporting measurements of higher valued (10 nF to 100 μF) ceramic-dielectric four-terminal-pair (4TP) capacitance standards over the 100-Hz to 100-kHz frequency range is described. The capacitance scaling system uses a commercial impedance (inductance–capacitance–resistance) meter and a single-decade inductive voltage divider as an impedance comparator. Four-terminal-pair capacitors in decade (10 : 1) steps from 10 nF to 100 F are measured. The system's 10 : 1 scaling error is determined using 100-pF and 1-nF air-dielectric 4TP capacitance standards with known capacitance and loss characteristics over frequency. This paper discusses the significant reductions in measurement uncertainty that were attained through the use of improved calibration standards and measurement method refinements. Details of the uncertainty analysis for a 10-nF capacitor (in the 100-Hz to 10-kHz frequency range) and verification data are presented.**

Index Terms—**Calibration, capacitance, four-terminal-pair 4TP capacitance, inductance–capacitance–resistance (LCR) meter, inductive voltage dividers (IVDs), uncertainty analysis.**

I. Introduction

A CAPACITANCE scaling system that was developed to support the measurement of a specific manufacturer's particular type of higher valued (10 nF to 100 μF) ceramic-dielectric four-terminal-pair (4TP) capacitance standards over the 100-Hz to 100-kHz frequency range has been described previously [1]. Two desirable aspects of this system are given as follows: 1) It is based mostly on commercially available instrumentation, and 2) the calibration procedure is largely automated. An investigation was conducted at the National Institute of Standards and Technology (NIST) to determine whether the measurement uncertainty of this system could be reduced to support the routine measurement of improved 4TP capacitance standards, either as they become commercially available or as recently developed at NIST to support generalized ac impedance metrology [2]. The chief goal of this investigation was to separate the relative contributions of measurement uncertainty due to the impedance comparator itself from those of the 100-pF and 1-nF 4TP air-dielectric capacitance standards that were used to calibrate it. A manual 4TP

capacitance bridge designed at NIST by Cutkosky in 1970 [3] was used to perform the investigation. This bridge can compare standards (with a short-term stability better than 1 part in 10^7) over the frequency range of 100 Hz to 20 kHz. The stability of the capacitor under test is crucial because of the complexity of the manual balancing process. A short-term stability should be better than 0.3 part in 10^6. Thus, it is not practical to calibrate commercial 4TP air- and ceramic-dielectric capacitors, which have temperature coefficients on the order of 10–30 parts in 10^6. As a result, commercially available nitrogen-dielectric capacitance standards were selected for use in the investigation.

II. Four-Terminal-Pair Scaling System Based on an *LCR* Meter

In 1997, Aoki and Yokoi [1] described a method of calibrating higher valued 4TP capacitors (10 nF to 100 μF) by using a 4TP impedance (*LCR*) meter, a decade inductive voltage divider (IVD), and two reference capacitors. The *LCR* meter is not only a stable impedance detector but is also a variable-frequency and variable-amplitude (current or voltage) source. The decade IVD is used to make a capacitor "appear" to be ten times larger by scaling the voltage applied to the sense terminals of the *LCR* meter. This allows the *LCR* meter to be used as a transfer device at approximately the same operating point (all but eliminating its linearity errors [4] and influence of the short-term stability) for capacitors that differ in values by a factor of ten.

A simplified diagram of the NIST capacitance scaling system is shown in Fig. 1. The scaling fixture controls the circuitry that was used to adjust signal inputs and connections of the 10 : 1 IVD that was used to scale values of capacitance from lower to higher. For a more detailed diagram of the scaling fixture, see [1]. The *LCR* meter and scaling fixture (interface and IVD) is self-calibrated by comparing two calibrated 4TP air capacitors with known frequency characteristics [5], [6]. This essentially establishes the scaling ratio of the fixture as a 1 : 1 and 10 : 1 interface. The scaling ratio self-calibration procedure and the uncertainty analysis are given in detail in the following section.

Four measurements are required to calibrate the 10 : 1 scaling ratio. First, a 100-pF capacitor is measured using a three-terminal (3T) capacitance bridge [9], i.e., C_{100pCB}. The impedance of this standard is then measured using the *LCR* meter, i.e., $Z_{100pLCR}$, with the 10 : 1 IVD switched into the circuit (this makes the 100-pF capacitor appear to have the impedance of a 1-nF capacitor). The impedance of a 1-nF standard is measured using the *LCR* meter without the IVD, i.e., Z_{1nLCR}. Finally, the capacitance of the 1-nF standard is measured using

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14. ABSTRACT

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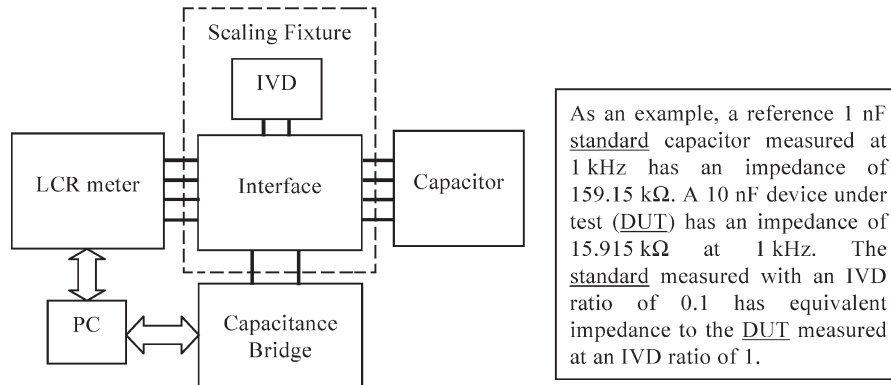
Fig. 1. Capacitance scaling system with a descriptive measurement example.

As an example, a reference 1 nF standard capacitor measured at 1 kHz has an impedance of 159.15 kΩ. A 10 nF device under test (DUT) has an impedance of 15.915 kΩ at 1 kHz. The standard measured with an IVD ratio of 0.1 has equivalent impedance to the DUT measured at an IVD ratio of 1.

the capacitance bridge, i.e., $C_{1\text{nCB}}$. The scaling ratio K is determined by

$$K = \frac{Z_{100\text{p}LCR}}{Z_{1\text{n}LCR}} \frac{C_{100\text{pCB}}}{C_{1\text{nCB}}}. \tag{1}$$

The capacitance scaling method that was described in [1] uses 4TP air capacitors. Although the 100-pF and 1-nF air capacitors have insignificant dissipation factors over the frequency range of interest, their capacitance is not particularly stable (typically 30 parts/10^6/°C). Over the course of the several months of measurements using the capacitance scaling system, we observed fluctuations in the 1-kHz capacitance of standard air capacitors on the order of 7 parts in 10^6. To discriminate between the uncertainty of the scaling system and the instability of the 4TP air standards, 3T nitrogen capacitors with values of 100 pF and 1 nF were introduced. At 1 kHz, the nitrogen capacitors varied by less than 0.1 part in 10^6. The nitrogen capacitors are connected to the 4TP connector at the scaling fixture using a coaxial adapter. In conclusion, the chosen measurement setup for calibration of the scaling ratio consists of a 3T capacitance bridge that is used to measure the 3T nitrogen capacitors at 100 Hz, 1 kHz, and 10 kHz and the 4TP air capacitors measured at 1 kHz. The 4TP capacitance at 100 kHz is obtained using an extrapolation method [5], [6]. These standard types were selected to achieve the best scaling ratio uncertainty performance when measuring capacitors with values of 100 pF and 1 nF over the frequency range from 100 Hz to 100 kHz. The 3T measurements do not have a significant effect on the uncertainty of calibrating 4TP capacitors. (See Fig. 2 for the simplified schematic of the measurement system and the parameter definitions.) The detailed analysis is performed in [8]. The influence of lead impedances on capacitance measurements is less than 1 part in 10^7 when measuring the 1-nF air capacitor at 1 kHz and the 1-nF nitrogen capacitor at 10 kHz.
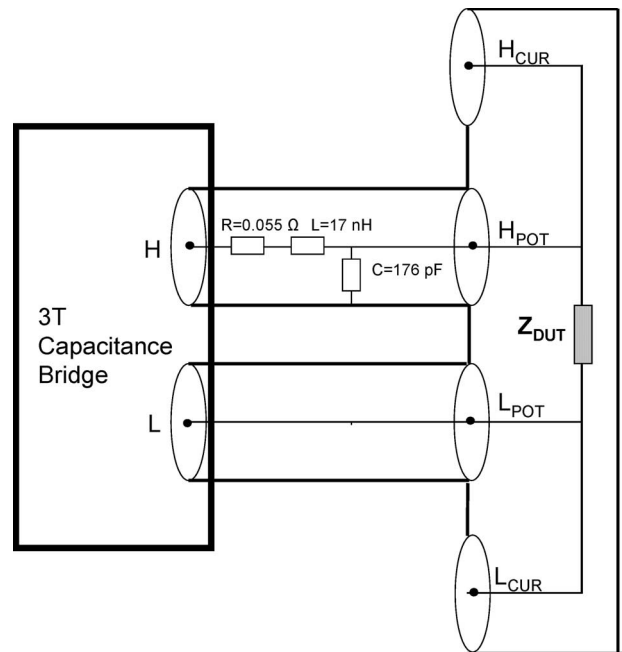

Fig. 2. Simplified diagram of the 3T capacitance bridge measurement of a 4TP air capacitor.

When using a 4TP *LCR* meter to measure a 3T nitrogen standard capacitor, both high current and high potential ports of the meter are connected to the high port of the standard. Both low ports of the meter are also tied together at the standard. All grounds are also connected.

The relative uncertainty of the scaling ratio $u_r(K)$ is given by (2), shown at the bottom of the page, where $\sigma(x)$ represents the standard deviation of measurement x, NL_{LCR} is the uncertainty contribution from the nonlinearity of the *LCR* meter and scaling fixture, and NL_{CB} is the uncertainty contribution from the nonlinearity of the capacitance bridge. Measurements were performed to determine the nonlinearity of the *LCR* meter.

$$u_r(K) = \sqrt{\left(\frac{\sigma\left(\frac{Z_{100\text{p}LCR}}{Z_{1\text{n}LCR}}\right)}{\left(\frac{Z_{100\text{p}LCR}}{Z_{1\text{n}LCR}}\right)}\right)^2 + \left(\frac{\sigma\left(\frac{C_{100\text{pCB}}}{C_{1\text{nCB}}}\right)}{\left(\frac{C_{100\text{pCB}}}{C_{1\text{nCB}}}\right)}\right)^2 + (\text{NL}_{LCR})^2 + (\text{NL}_{\text{CB}})^2} \tag{2}$$

TABLE I
FREQUENCY DEPENDENCE OF SCALING RATIO UNCERTAINTY $u_r(K)$ (IN PARTS IN 10^6)

Frequency	$u_r(K)$ using air capacitors	$u_r(K)$ using nitrogen capacitors
100 Hz	20	7
1 kHz	10	2
10 kHz	10	2

Using capacitors that varied from nominal, which is as much as 100 parts in 10^6, the *LCR* readings were in error by less than one part in 10^6, supporting previous studies [4]. Similarly, the nonlinearity of the capacitance bridge was found to be less than one part in 10^6.

Based on (2), the uncertainty of the scaling ratio $u_r(K)$ is typically two parts in 10^6 using nitrogen capacitors and 10 parts in 10^6 using air capacitors at 1 kHz.

The IVD used in the scaling fixture should be very stable. To separate the influence of the interface from the IVD, we replaced it with a commercial IVD and observed the same level of variations in K (two parts in 10^6 using nitrogen capacitors). This indicates that the stabilities of the interface, the *LCR* meter, and the capacitors are the main contributors to the uncertainty of the scaling ratio calibration. To determine whether this instability is time dependent, we shortened the measurement process from 30 to 5 min. The standard deviation of the scaling ratio was reduced by nearly an order of magnitude to approximately 0.3 part in 10^6.

Scaling ratio uncertainty tests were performed between 100 Hz and 10 kHz using 100-pF and 1-nF air and nitrogen capacitors, and the results are presented in Table I. The capacitance scaling system measures the scaling ratio at 100 kHz, and air capacitors have the relative uncertainty of 15 parts in 10^6. This uncertainty was obtained using (2).

III. NANOFARAD CAPACITOR MEASUREMENT

Once the scaling ratio is determined, the system can be used to calibrate capacitors. A 1-nF reference capacitor is measured using the capacitance bridge, i.e., C_{1nrCB}, and using the *LCR* meter with the IVD, i.e., Z_{1nrLCR}. Then, a 10-nF device under test (DUT) Z_{DUT10n} is measured using the *LCR* meter without the IVD, i.e., Z_{10nLCR}. These measurements are used to find the impedance of the DUT

$$Z_{DUTn} = K \frac{1}{\omega C_{1nrCBR}} \frac{Z_{10nLCR}}{Z_{1nrLCR}}. \tag{3}$$

TABLE II
TEN-NANOFARAD CAPACITOR MEASUREMENTS AND THE UNCERTAINTY LEVELS (IN PARTS IN 10^6) AT SEVERAL FREQUENCIES

	Deviation from NIST 4TP Bridge Measurement			Measurement Uncertainty Coverage Factor ($k = 1$)		
Frequency (kHz)	0.1	1	10	0.1	1	10
NIST 4TP Bridge	0	0	0	0.3	<0.1	0.2
Capacitance Scaling System	19	1.9	1.2	23*	4*	6*

Note that the frequency characteristic of the 1-nF reference capacitor is assumed to be flat.

The relative uncertainty $u_r(Z_{10nLCR})$ is given by (4), shown at the bottom of the page.

The same notation as in (2) is followed. Note that the measurement of C_{1nrCB} has both type A (standard deviation of the measurements) and type B (u_{1nrCB}) error components. A numerical estimation of the 10-nF impedance uncertainty is given as

$$\frac{\partial Z_{DUT10n}}{Z_{DUTn}} \approx \frac{\partial C_{DUTn}}{C_{DUTn}}$$
$$= 10^{-6} \sqrt{2^2 + 6^2 + 1^2 + 0.1^2 + 1^2}$$
$$= 6.5 \times 10^{-6}. \tag{5}$$

The actual fluctuations of the measured 10-nF nitrogen capacitance standard produced a standard deviation of four parts in 10^6, which is in good agreement with (5).

A 10-nF nitrogen-dielectric capacitor with a short-term stability better than 0.1 part in 10^6 was measured using the NIST 4TP bridge and the capacitance scaling system. A summary of the results is shown in Table II.

The results in Table II show excellent agreement between the NIST manual bridge and the capacitance scaling system at 1 and 10 kHz using stable 1-nF and 10-nF 3T nitrogen capacitors. The agreement at 100 Hz is not as good but is still within 20 parts in 10^6. The results are considerably better than those obtained using 4TP air capacitors, which is due to their inherent instability.

The capacitance scaling system was originally designed [1] to use 4TP air standards as references due to their known frequency characteristics [5]. To determine the influence of the frequency dependence of the nitrogen capacitors on determination of the 10-nF capacitance, a 1-nF nitrogen capacitor was

$$u_r(Z_{DUT10n}) = \sqrt{\left(\frac{\sigma K}{K}\right)^2 + \left(\frac{\sigma\left(\frac{Z_{10nLCR}}{Z_{1nrLCR}}\right)}{\left(\frac{Z_{10nLCR}}{Z_{1nrLCR}}\right)}\right)^2 + (NL_{LCR})^2 + \left(\frac{\sigma C_{1nrCB}}{C_{1nrCB}}\right)^2 + (u_{1nrCB})^2} \tag{4}$$

measured to vary by less than 2 parts in 10^6 in the range from 100 Hz to 10 kHz. This variation influenced the results for the 10-nF capacitor less than 1 part in 10^6. This result indicates that nitrogen-dielectric capacitors should be used as references at frequencies up to 10 kHz, whenever possible. However, air capacitors should be used as references for the 100-kHz range because the uncertainty of the frequency dependence of nitrogen-dielectric capacitors is larger than the temperature coefficient instability of the 4TP air standards. The extension to 100 kHz is based on the fact that frequency characteristics of a 1-nF reference capacitor are known and that scaling ratio is obtained using well-characterized 100-pF and 1-nF reference capacitors (calibration performed using a network analyzer [6], [7]).

IV. CONCLUSION

This evaluation of an *LCR*-based capacitance scaling system has shown its potential for calibrating stable capacitors in the 100-pF to 10-nF range. Preliminary results indicate that the system is capable of measuring capacitance and dissipation factor for higher valued 4TP solid-dielectric standards (up to 100 μF) over a wide frequency range (100 Hz to 100 kHz). The stability of these measurements is limited by the stability of the DUT. However, further tests are required to determine the uncertainties of the system over this range. For further information on this work, see [8].

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